Influence of losses on the wave-particle duality

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We theoretically investigate the influence of losses on the wave-particle duality in a Mach-Zehnder interferometer. Balanced losses (equal loss on the two paths) have no effect on the visibility and predictability (also the duality relation), while unbalanced losses do have influence on them. If the unbalanced losses occur inside the interferometer, the losses have an effect on both the visibility and predictability, while the losses have no effect on the duality relation. If the unbalanced losses occur after the interferometer, the losses have no effect on the visibility, but do have an effect on the predictability (also the wave-particle duality), which can lead to a result that the duality relation exceeds 1, $P^2 + V^2 > 1$. The influence of losses can be eliminated by exchanging the two detectors or the two inputs (one photon and the vacuum) of the interferometer and then averaging the two results. Consequently, we have the visibility, predictability, and duality relation $P^2 + V^2 \leqslant 1$ independent of the unbalanced losses. The obtained P and V for the unbalanced losses in the two paths do not represent the original predictability and visibility, and the result of $P_D^2 + V_D^2 > 1$ does not mean the "violation" of the original duality.

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I. INTRODUCTION

In 1928, Bohr proposed the important principle of complementarity [1], which lies at the heart of quantum mechanics. This complementarity emphasizes that the quantum systems possess properties that are equally real but mutually exclusive [2,3], such as the wave-particle duality. One can observe wavelike or particlelike behavior of the particle through different measuring devices. The earlier discussion on the complementarity was based on Young's double-slit experiment with a light beam which contains many photons [4,5]. In 1979, Wootters and Zurek quantified the wave-particle duality [6]. The first inequality, $P^2 + V^2 \leqslant 1$, with P the predictability of the particle (photon) passing along the two paths and V the visibility of the interference pattern behind the standard Mach-Zehnder interferometer (MZI), is theoretically derived by Jaeger *et al.* [7] and Englert [8], which can be used to quantify the wave-particle duality for a single particle. In Ref. [8], the second duality inequality $D^2 + V^2 \leqslant 1$ for a single particle is introduced, when the interferometer (MZI) is supplemented with a which-way detector (WWD). In this inequality, the distinguishability D represents *a posteriori* which-way knowledge after the particle interacted with the WWD, and V is the fringe visibility. Here we would emphasize that the definitions of the two inequalities are different. The predictability is the difference between the probabilities (w_1 and w_2) that the particle takes one way and the other, $P = |w_1 - w_2|$. For the distinguishability, the which-way information is stored in the which-way detector (WWD) and can be read out from the detector. After the photon's interaction with the which-way detector, we can get the final detector state ρ_D. Assuming $|W\rangle$ is an eigenvector of the detector, the probabilities of photons following path 1 and path 2 are $\langle W|\,\rho_D^{(1)}\,|W\rangle$ and $\langle W|\,\rho_D^{(2)}\,|W\rangle$. If $\langle W|\,\rho_D^{(1)}\,|W\rangle > \langle W|\,\rho_D^{(2)}\,|W\rangle$ (or $\langle W|\,\rho_D^{(2)}\,|W\rangle > \langle W|\,\rho_D^{(1)}\,|W\rangle$), we can best guess the particle goes through path 1 (or path 2). The "likelihood for guessing the right way" is given by $L = \sum_W \max\{\langle W|\,\rho_D^{(1)}\,|W\rangle\,,\langle W|\,\rho_D^{(2)}\,|W\rangle\}$. If we can find the optimized eigenvector $|W\rangle$, which results in the largest value of L, $L_{\max}=\frac{1}{2}\,(1 + D)$, the distinguishability is quantified as $D = 2L_{\max} - 1$ [8]. Both predictability and distinguishability are well defined in Ref. [8]. The experiments reported in Refs. [9,10] demonstrate the second inequality and many experiments [11–17] verify the first inequality of the duality. The inequality of the duality is valid even in a Wheeler's delayed-choice experiment [18,19] and has been confirmed experimentally for a single particle in 2007 and 2008 [20,21]. However, the "distinguishability" defined in Refs. [20,21] is actually different from the distinguishability defined in Ref. [8]. In order to distinguish these two different definitions of distinguishability, we call the distinguishability in Refs. [20,21] "which-way knowledge."

Recently, a new optical device called the quantum beam splitter (QBS) [22,23] is proposed by adding other degrees of freedom of the single particle, such as the polarization of a photon. It is theoretically declaimed that the particlelike and wavelike behaviors of the particle can be tested at the same time. The experiments with the QBS were reported in Refs. [24–26] and the wave-particle duality is still satisfied. However, the simultaneous detection on the particle and wave behaviors is not achieved. Recently in Ref. [27] (a single photon with polarizations as the two additional degrees of freedom), an interesting result, $P^2 + V^2 > 1$, was reported, where it was declaimed that this result is due to the interference between the particle and wave behaviors. In Ref. [27], one certain base (the polarizer set at an angle β) for the QBS is chosen for the detection; that is to say, the photon is projected into this base, while the photons in the orthogonal base

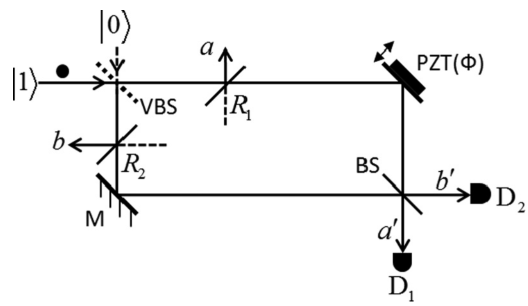

FIG. 1. Losses inside the Mach-Zehnder interferometer. A single input particle is split by a variable beam splitter (VBS) with adjustable reflectivity R. The loss in the interferometer is modeled by the beam splitter, with reflectivity R_1 (R_2). PZT stands for the piezoelectric transducer and M stands for a normal mirror. Then the particle is recombined with a 50:50 beam splitter (BS) and detected by detectors 1 (D_1) and 2 (D_2). a,b,a' and b' stand for the path.

$(\beta + \frac{\pi}{2})$ are not recorded, which is equivalent to the photon loss.

In the above works, the influence of losses on the wave-particle duality is not discussed. However, losses are inevitable in any experiment, and the study on the losses is necessary. In this paper, we studied the duality relation for a single particle in the standard MZI setup with losses. We consider two kinds of losses, one inside the MZI, and the other behind the MZI but before the detection in Secs. II and III, respectively. Section IV is a summary. In the Appendix, we discuss the influence of losses on the duality used in Ref. [21].

II. LOSSES INSIDE THE INTERFEROMETER

We discuss the influence of losses on the wave-particle duality in the standard MZI (see Fig. 1) and analyze the first inequality, following Ref. [8]. A single input particle is split by a variable beam splitter (VBS) with adjustable reflectivity R. In order to simulate the losses, we use two beam splitters (BSs) with reflectivities R_1 and R_2 ($0 \leqslant R_1, R_2 < 1$) in the two paths, respectively. The piezoelectric transducer (PZT) is used to adjust the relative phase between the two paths. At last the particle is recombined with a 50:50 beam splitter (BS) and detected by detectors 1 (D_1) and 2 (D_2), and a (or b) indicates the path where the loss occurs. First we assume that both detectors are perfect; the situation with the imperfect detectors will be discussed in the following.

The wavelike behavior can be described by the visibility of the interference fringe pattern after the last 50:50 BS [8],

$$V = \frac{p_{\max} - p_{\min}}{p_{\max} + p_{\min}}, \tag{1}$$

where p is the probability that the particle follows path a' (b') and is detected on detector 1 (2). The maximum (max) and minimum (min) value are obtained by scanning the phase ϕ.

For this device with losses inside the interferometer, the final state after the BS is

$$|\psi_{f1}\rangle = A_1|1\rangle_{a'} + A_2|1\rangle_{b'} + A_3|1\rangle_a + A_4|1\rangle_b, \tag{2}$$

with

$$A_1 = \sqrt{\frac{(1-R)(1-R_1)}{2}}e^{i\phi} - \sqrt{\frac{R(1-R_2)}{2}};$$
$$A_2 = \sqrt{\frac{(1-R)(1-R_1)}{2}}e^{i\phi} + \sqrt{\frac{R(1-R_2)}{2}};$$
$$A_3 = \sqrt{R_1(1-R)}; \quad A_4 = \sqrt{RR_2}.$$

The probability p_1 that the particle is detected on detector 1 is

$$p_1 = {}_{a'}\langle 1 \mid \psi_{f1}\rangle\langle\psi_{f1} \mid 1\rangle_{a'} = |A_1|^2. \tag{3}$$

The interference pattern can be observed by scanning ϕ. According to Eq. (1), the *a priori* fringe visibility is

$$V = \frac{2\sqrt{R(1-R)(1-R_1)(1-R_2)}}{(1-R)(1-R_1) + R(1-R_2)}. \tag{4}$$

We can also obtain the same visibility if we detect the particle by detector 2.

For the predictability, we need to remove the last BS and detect the probabilities of particle on paths a' and b'. The difference between the two probabilities denotes the which-way information,

$$P = \frac{|w_1 - w_2|}{w_1 + w_2}, \tag{5}$$

where w_1 and w_2 stand for the probabilities of the particle detected on detectors 1 and 2, respectively. The final state after removing the BS becomes

$$|\psi_{f2}\rangle = B_1\,|1\rangle_{a'} + B_2\,|1\rangle_{b'} + B_3\,|1\rangle_a + B_4\,|1\rangle_b\,, \tag{6}$$

with $B_1 = e^{i\phi}\sqrt{(1-R)(1-R_1)}$; $B_2 = \sqrt{R(1-R_2)}$; $B_3 = \sqrt{(1-R)R_1}$; $B_4 = \sqrt{RR_2}$.

The probabilities for taking either one of the two ways are $w_1 = |B_1|^2$ and $w_2 = |B_2|^2$. They are detected on detectors 1 and 2, respectively. According to Eq. (5), the predictability of the ways through the interferometer is

$$P = \frac{|(1-R)(1-R_1) - R(1-R_2)|}{(1-R)(1-R_1) + R(1-R_2)}. \tag{7}$$

It is easy to verify the wave-particle duality relation:

$$P^2 + V^2 = 1. \tag{8}$$

It is clear that for the losses inside the interferometer, the probabilities of particles passing through the two paths are reduced by the amount of $(1 - R_{1,2})$, which can be found in Eqs. (4) and (7). Therefore, the unbalanced losses inside the MZI will influence the visibility and predictability. In Fig. 2, we plot P^2 and V^2 versus R for $R_1 = 0.4$ and $R_2 = 0.6$, where we also plot the corresponding curves without losses. However, the losses have no influence on the wave-particle duality relation for the device with losses inside the MZI (before the last BS), as shown in Eq. (8).

III. LOSSES AFTER THE INTERFEROMETER

After the particle passing through the second BS in the MZI (see Fig. 3), photon loss may be caused by the paths from MZI to the detectors or the detectors themselves. These losses can still be simulated by two BSs, as depicted in Fig. 3. The loss rate is R_1' (R_2') with $0 \leqslant R_1'(R_2') < 1$.

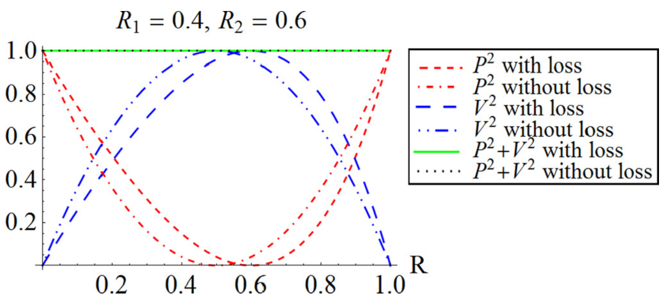

FIG. 2. (Color online) P^2 and V^2 versus R for $R_1 = 0.4$ and $R_2 = 0.6$. The red and blue lines stand for P^2 and V^2, respectively. The green solid line and black dotted line correspond to $P^2 + V^2$ with and without losses, respectively.

Similarly, the visibility is expressed as

$$V_D = \frac{p'_{\max} - p'_{\min}}{p'_{\max} + p'_{\min}}, \tag{9}$$

where p' is the probability of particles detected on detector 1 or 2. The final state after the BSs of R'_1 and R'_2 is

$$|\psi'_{f1}\rangle = A'_1\,|1\rangle_{a'} + A'_2\,|1\rangle_{b'} + A'_3\,|1\rangle_a + A'_4\,|1\rangle_b\,, \tag{10}$$

with

$$A'_1 = \sqrt{\frac{1-R'_1}{2}}(e^{i\phi}\sqrt{1-R} - \sqrt{R});$$
$$A'_2 = \sqrt{\frac{1-R'_2}{2}}(e^{i\phi}\sqrt{1-R} + \sqrt{R});$$
$$A'_3 = \sqrt{\frac{R'_1}{2}}(e^{i\phi}\sqrt{1-R} - \sqrt{R});$$
$$A'_4 = \sqrt{\frac{R'_2}{2}}(e^{i\phi}\sqrt{1-R} + \sqrt{R}).$$

The probability p'_1 that the particle is detected on detector 1 is

$$p'_1 = {}_{a'}\langle 1|\psi'_{f1}\rangle\langle\psi'_{f1}|1\rangle_{a'} = |A'_1|^2. \tag{11}$$

By adjusting the relative phase ϕ, the maximum and minimum values can be obtained. According to Eq. (9), the fringe visibility is

$$V_D = 2\sqrt{R(1-R)}. \tag{12}$$

We can obtain the same result if we detect the particle by detector 2.

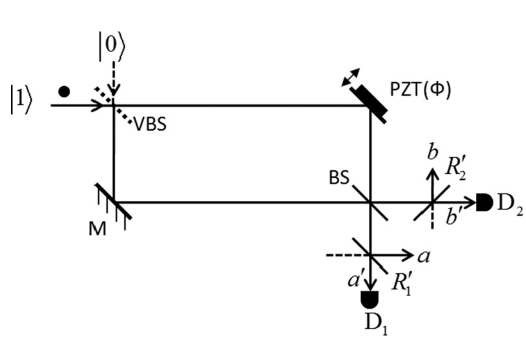

FIG. 3. Losses after Mach-Zehnder interferometer. The loss rate is R'_1 (R'_2). The other elements are same as the case in Fig. 1.

The visibility is calculated with Eq. (9), which is determined by the maximizing and minimizing values of the counting rates, and the weight of R'_1 (R'_2) can be eliminated during the computational process. Consequently, the losses outside the MZI (R'_1 and R'_2) has no influence on the visibility as shown in Eq. (12).

For the predictability, it is given by

$$P_D = \frac{|w'_1 - w'_2|}{w'_1 + w'_2}, \tag{13}$$

which is obtained by removing the last 50:50 BS, where w'_1 and w'_2 stand for the probabilities of the particles detected on detectors 1 and 2, respectively. The final state becomes

$$|\psi'_{f2}\rangle = B'_1|1\rangle_{a'} + B'_2|1\rangle_{b'} + B'_3|1\rangle_a + B'_4|1\rangle_b, \tag{14}$$

with

$$B'_1 = e^{i\phi}\sqrt{(1-R)(1-R'_1)};\quad B'_2 = \sqrt{R(1-R'_2)};$$
$$B'_3 = e^{i\phi}\sqrt{(1-R)R'_1};\quad B'_4 = \sqrt{RR'_2}.$$

The probabilities for particles taking either one of the two paths (paths a' and b') and being detected, respectively, on detectors 1 and 2 are $w'_1 = |B'_1|^2$ and $w'_2 = |B'_2|^2$. According to Eq. (13), the predictability is

$$P_D = \frac{|(1-R)(1-R'_1) - R(1-R'_2)|}{(1-R)(1-R'_1) + R(1-R'_2)}, \tag{15}$$

which depends on R'_1 and R'_2. As the detecting probabilities of the two detectors depend separately on the losses R'_1 and R'_2, the predictability depends on the losses. Thus the wave-particle duality relation obtained from the two detectors is

$$P_D^2 + V_D^2 = \left[\frac{|(1-R)(1-R'_1) - R(1-R'_2)|}{(1-R)(1-R'_1) + R(1-R'_2)}\right]^2 + 4R(1-R). \tag{16}$$

In order to observe the influence of losses on duality relation clearly, we plot $P_D^2 + V_D^2$ as a function of R'_1 and R'_2 for different R (the reflectivity of the VBS) in Fig. 4. Note that the values of $P_D^2 + V_D^2$ for R and $1-R$ are symmetrical with respect to the diagonal line of the figures. It is clear that $P_D^2 + V_D^2$ can be larger than 1 for some certain values of R'_1 and R'_2. For example, when the reflectivity of the VBS is $R = 0.5$ and the loss rates are $R'_1 = 0.2$, $R'_2 = 0.8$, we have $P_D^2 + V_D^2 = 1.36$. For $R'_1 = 0.4$ and $R'_2 = 0.6$, we plot $P_D^2 + V_D^2$ as a function of R in Fig. 5, where we have $P_D^2 + V_D^2 > 1$ for some certain values of R. Please note that $P_D^2 + V_D^2 > 1$ can be eliminated with balanced losses ($R'_1 = R'_2$), as we have $P_D = |1-2R|$, $V_D = 2\sqrt{R(1-R)}$, and $P_D^2 + V_D^2 = 1$ from Eq. (16).

When unbalanced loss occurs, the obtained probability for calculating P_D is different from the probabilities without the losses. That is to say, the P_D obtained with loss is not the original P defined in Ref. [8]. It is obvious that the unbalanced losses affect the guess on which path the photon takes, which makes the determination on the predictability different from that without losses. Although we did not make calculations for the situation of a quantum beam splitter in [27], the losses

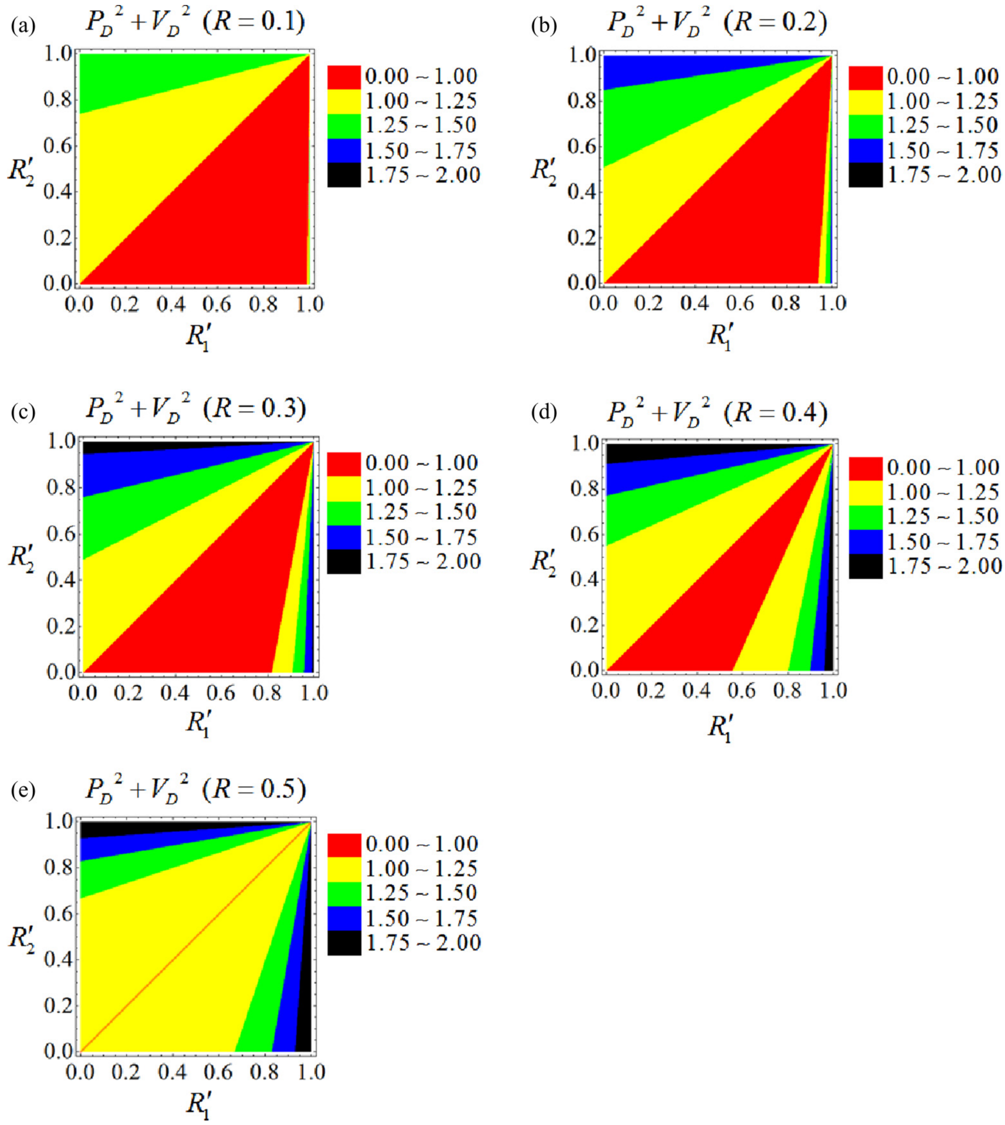

FIG. 4. (Color online) $P_D^2 + V_D^2$ as a function of R_1' and R_2'. (a)–(e) correspond to the cases with $R = 0.1 - 0.5$. $P_D^2 + V_D^2$ can be larger than 1 for some certain values of R_1' and R_2'.

discussed here may provide a reference on the experimental result in Ref. [27].

Next let us consider how to correct the result of $P_D^2 + V_D^2 > 1$ obtaining the original $P^2 + V^2 \leqslant 1$ defined in [8]. As

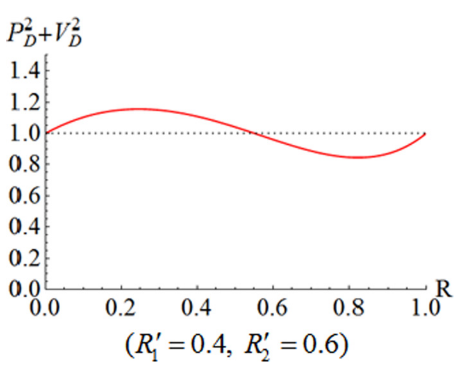

FIG. 5. (Color online) The red solid line describes $P_D^2 + V_D^2$ as a function of R for $R_1' = 0.4$ and $R_2' = 0.6$. The dashed line stands for the maximum standard line of duality relation, $P^2 + V^2 = 1$.

mentioned above, it can be eliminated by the balanced losses. If the losses are caused by the detectors, we can exchange the two detectors and detect the particle probabilities on the two detectors again, then take the average probabilities as the final result. Since the losses after the MZI have no effect on the visibility, we only need to average the predictabilities. The particle probabilities on detectors 1 and 2 after exchanging the two detectors are

$$w_{1e}' = (1 - R)(1 - R_2'),\ \ w_{2e}' = R(1 - R_1'). \tag{17}$$

The averages of the particle probabilities detected on detectors 1 and 2 are

$$\begin{aligned} \bar{w}_1 &= \frac{w_1' + w_{1e}'}{2} = \frac{1}{2}(1 - R)\left[(1 - R_1') + (1 - R_2')\right], \\ \bar{w}_2 &= \frac{w_2' + w_{2e}'}{2} = \frac{1}{2}R\left[(1 - R_1') + (1 - R_2')\right]. \end{aligned} \tag{18}$$

By using the average probabilities $\bar{w}_1$ and $\bar{w}_2$, the predictability becomes

$$P_D = \frac{|\bar{w}_1 - \bar{w}_2|}{\bar{w}_1 + \bar{w}_2} = |1 - 2R|\,. \tag{19}$$

The ordinary wave-particle duality relation $P_D^2 + V_D^2 = 1$ is recovered. By this method, we can eliminate the influence of the losses on the wave-particle duality relation. If the losses are caused by the paths between the MZI and the detectors, we exchange the input paths and the subscripts of the two detectors. We detect the particle probabilities on the two detectors again, and regard the average probabilities as the final result. For the predictability, the particle probabilities on detectors 1 and 2 after removing the last 50:50 BS and exchanging the input paths are

$$w''_{1e} = (1-R)(1-R'_2), w''_{2e} = R(1-R'_1). \tag{20}$$

The averages of the particle probabilities detected on detectors 1 and 2 are

$$\bar{w}'_1 = \frac{w'_1 + w''_{1e}}{2} = \frac{1}{2}(1-R)[(1-R'_1)+(1-R'_2)],$$
$$\bar{w}'_2 = \frac{w'_2 + w''_{2e}}{2} = \frac{1}{2}R[(1-R'_1)+(1-R'_2)]. \tag{21}$$

The predictability by using the average probabilities $\bar{w}'_1$ and $\bar{w}'_2$ becomes

$$P_D = \frac{|\bar{w}'_1 - \bar{w}'_2|}{\bar{w}'_1 + \bar{w}'_2} = |1 - 2R|. \tag{22}$$

Consequently, the influence of losses caused by the paths from MZI to the detectors can be eliminated. The ordinary wave-particle duality relation $P_D^2 + V_D^2 = 1$ is recovered.

IV. CONCLUSION

We have studied the influence of losses on the duality relation in a standard MZI setup and discussed how to eliminate it. If the losses appear inside the MZI, it takes effect on the visibility and predictability, but has no influence on the wave-particle duality relation. If the losses appear behind the MZI, the losses have a great influence on the wave-particle duality relation. Unbalanced losses after the MZI can lead to $P_D^2 + V_D^2 > 1$ (the "violation" of the ordinary duality relation). The obtained P_D and V_D for the unbalanced losses in the two paths do not represent the original predictability and visibility defined in Ref. [8]. The influence of losses on the visibility, predictability, and duality relation ($P_D^2 + V_D^2 > 1$) can be eliminated by exchanging the two detectors or the two inputs (one photon and the vacuum) of the interferometer and then averaging the two results.

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APPENDIX: THE INFLUENCE OF LOSSES ON THE DUALITY FOR THE DEVICE IN REF. [21]

The device in Ref. [21] is the same as that which we discussed above, except the locations of VBS and 50:50BS are exchanged. If there is no loss, the dualities for these two devices are the same [11]. But if we consider the losses, the results for these two devices are different.

In Ref. [21], the definition of visibility is the same as that above, and the which-way knowledge is defined as

$$K = \frac{K_1 + K_2}{2}, \tag{A1}$$

$$K_1 = \left.\frac{|p_{21} - p_{22}|}{p_{21} + p_{22}}\right|_{\text{path 1 blocked}},$$
$$K_2 = \left.\frac{|p_{11} - p_{12}|}{p_{11} + p_{12}}\right|_{\text{path 2 blocked}}, \tag{A2}$$

where p_{ij} is the probability that the particle follows path i (the other path is blocked) and is detected on detector j.

1. Losses inside the MZI

First, we analyze the case that the losses are inside the MZI (see Fig. 6). The final state before the detectors is

$$|\psi_f\rangle = C_1|1\rangle_{a'} + C_2|1\rangle_{b'} + C_3|1\rangle_a + C_4|1\rangle_b, \tag{A3}$$

with

$$C_1 = e^{i\varphi}\sqrt{\frac{(1-R)(1-R_1)}{2}} - \sqrt{\frac{R(1-R_2)}{2}};$$
$$C_2 = e^{i\varphi}\sqrt{\frac{R(1-R_1)}{2}} + \sqrt{\frac{(1-R)(1-R_2)}{2}};$$
$$C_3 = \sqrt{\frac{R_1}{2}};\ C_4 = \sqrt{\frac{R_2}{2}}.$$

The interference pattern can be observed by scanning ϕ. According to Eq. (1), the visibilities detected by detectors 1 and 2 are

$$V_1 = \frac{2\sqrt{R(1-R)(1-R_1)(1-R_2)}}{(1-R)(1-R_1) + R(1-R_2)}, \tag{A4}$$

$$V_2 = \frac{2\sqrt{R(1-R)(1-R_1)(1-R_2)}}{R(1-R_1) + (1-R)(1-R_2)}, \tag{A5}$$

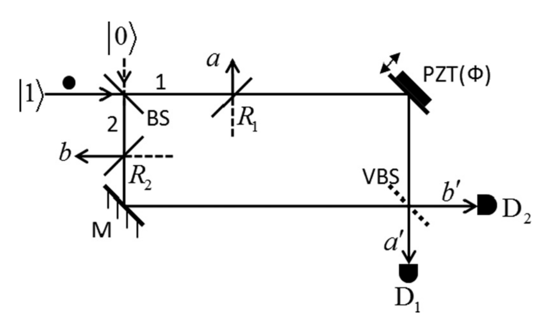

FIG. 6. Losses inside the MZI. This device is same as Fig. 1 except that the VBS and BS are switched.

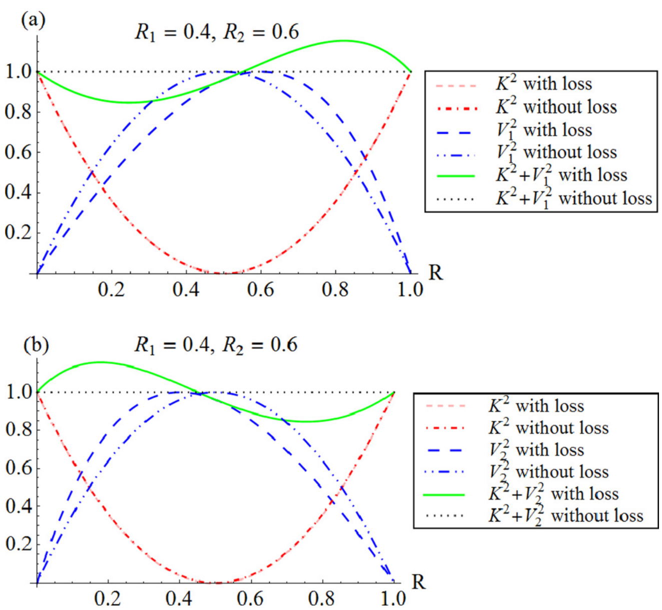

FIG. 7. (Color online) K^2 and $V_{1(2)}^2$ versus R for $R_1 = 0.4$ and $R_2 = 0.6$.

respectively. For the which-way knowledge, we need to block one path according to Ref. [21]. The final state becomes

$$|\psi_{f1}\rangle = \sqrt{1-R_2}(\sqrt{1-R}|1\rangle_{b'} - \sqrt{R}|1\rangle_{a'}) + \sqrt{R_2}|1\rangle_b \quad \text{(path 1 blocked)}, \tag{A6}$$

$$|\psi_{f2}\rangle = e^{i\varphi}\sqrt{1-R_1}(\sqrt{1-R}|1\rangle_{a'} + \sqrt{R}|1\rangle_{b'}) + \sqrt{R_1}|1\rangle_a \quad \text{(path 2 blocked)}. \tag{A7}$$

According to Eqs. (A1) and (A2), the which-way knowledge is

$$K = \frac{K_1 + K_2}{2} = |1 - 2R|, \tag{A8}$$

with

$$K_1 = K_2 = |1 - 2R|.$$

The influence of losses can be eliminated by balancing the losses. Otherwise, the losses have an effect on the visibility and which-way knowledge and the duality $K^2 + V^2$ may be bigger than 1. For $R_1 = 0.4$ and $R_2 = 0.6$, we plot $K^2 + V_{1(2)}^2$ as a function of R in Fig. 7, where we have $K^2 + V_{1(2)}^2 > 1$ for some certain values of R.

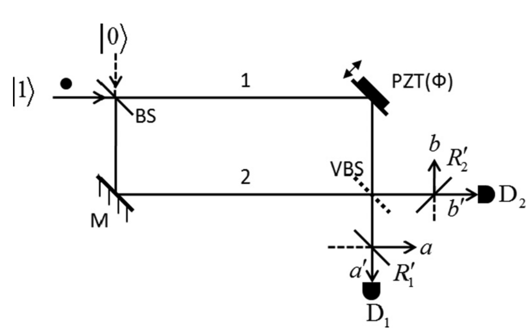

FIG. 8. Losses after the MZI. This device is the same as Fig. 3 except that the VBS and BS are switched.

2. Losses after the MZI

Similarly, for the case that the losses are outside the MZI (see Fig. 8), the final state before detectors is

$$|\psi_f'\rangle = C_1'|1\rangle_{a'} + C_2'|1\rangle_{b'} + C_3'|1\rangle_a + C_4'|1\rangle_b, \tag{A9}$$

with

$$C_1' = \sqrt{\frac{1-R_1'}{2}}(e^{i\varphi}\sqrt{1-R} - \sqrt{R});$$
$$C_2' = \sqrt{\frac{1-R_2'}{2}}(e^{i\varphi}\sqrt{R} + \sqrt{1-R});$$
$$C_3' = \sqrt{\frac{R_1'}{2}}(e^{i\varphi}\sqrt{1-R} - \sqrt{R});$$
$$C_4' = \sqrt{\frac{R_2'}{2}}(e^{i\varphi}\sqrt{R} + \sqrt{1-R}).$$

According to Eq. (9), the visibility detected by detectors 1 and 2 are the same:

$$V' = 2\sqrt{R(1-R)}. \tag{A10}$$

For the which-way knowledge, we block one path and the final states become

$$|\psi_{f1}'\rangle = -\sqrt{R}(\sqrt{1-R_1'}|1\rangle_{a'} + \sqrt{R_1'}|1\rangle_a) + \sqrt{1-R}(\sqrt{1-R_2'}|1\rangle_{b'} + \sqrt{R_2'}|1\rangle_b) \quad \text{(path 1 blocked)}, \tag{A11}$$

$$|\psi_{f2}'\rangle = e^{i\varphi}\sqrt{1-R}(\sqrt{1-R_1'}|1\rangle_{a'} + \sqrt{R_1'}|1\rangle_a) + e^{i\varphi}\sqrt{R}(\sqrt{1-R_2'}|1\rangle_{b'} + \sqrt{R_2'}|1\rangle_b) \quad \text{(path 2 blocked)}. \tag{A12}$$

According to Eqs. (A1) and (A2), the which-way knowledge is

$$K' = \frac{K_1' + K_2'}{2} = \frac{1}{2}\frac{|R(1-R_1') - (1-R)(1-R_2')|}{R(1-R_1') + (1-R)(1-R_2')} + \frac{1}{2}\frac{|(1-R)(1-R_1') - R(1-R_2')|}{(1-R)(1-R_1') + R(1-R_2')}, \tag{A13}$$

with

$$K_1' = \frac{|R(1-R_1') - (1-R)(1-R_2')|}{R(1-R_1') + (1-R)(1-R_2')},$$
$$K_2' = \frac{|(1-R)(1-R_1') - R(1-R_2')|}{(1-R)(1-R_1') + R(1-R_2')}.$$

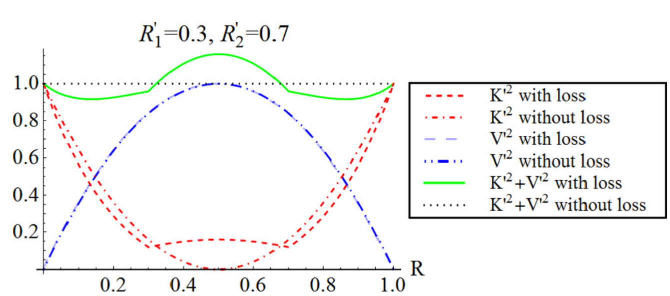

FIG. 9. (Color online) K'^2 and V'^2 versus R for $R_1' = 0.3$ and $R_2' = 0.7$.

We find the unbalanced losses have an effect on the which-way knowledge as it has influence on guessing which path the photon takes. The duality $K'^2 + V'^2$ may be bigger than 1 for the unbalanced losses. For $R_1' = 0.3$ and $R_2' = 0.7$, we plot $K'^2 + V'^2$ as a function of R in Fig. 9, which shows $K'^2 + V'^2 > 1$ for some certain values of R.

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